THE USE OF CONFIGURATION INTERACTION METHOD FOR DESCRIBING "FINE" - SPLITTING EFFECTS IN THE BOUND TWO-QUARK SYSTEM

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Abstract

The screened quasi-relativistic potential is used for describing spin-orbit splitting in ^3P_J waves of quark-antiquark system. Fermi-Breit equation is solved numerically in configuration interaction approximation. This approximation takes into account the mixing of wave functions up to fifth order and corrects substantially perturbation calculations. The nature of potentials Lorentz transformation property is elucidated. Very good quantitative results for $b\overline{b}$ and $c\overline{c}$ quarkonia and quite acceptable qualitative characteristics as well as for systems with unequal masses are obtained for $u\overline{u}$.

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Today it seems evident that quark potential model gives a rather good description of spin-averaged mass spectrum of hadrons, considered as a system of quarks (see e.g. [1] and literature cited there). Usually non-relativistic Cornell as well as oscillator potential with Coulumb-like one-gluon exchange or other power-law confinement terms is used. In this work we try to extend this approach to incorporate the second order spin-terms in two-quark Fermi-Breit equation for evaluating the spin-orbit splitting. Instead of calculating as usual spin-terms in first-order perturbation approximation the expansion of the total wave function into a basic set of non-perturbative solutions up to the fifth order of configurationally interacting states is carried out.

The influence of a specific form of potential on a spin-effects (like Cornel, oscillator, screened etc.) which generally give the same results for spin-averaged data, the way of accounting of both long-range non-perturbative and short-range (perturbative) parts in these calculations, the Lorentz structure of different parts of potentials all these questions remain to be unsolved yet.

Our main goal is to clarify some aspects of these questions in framework of configuration interaction (CI) approach [2]. This method does not need the assumption that the coupling constant is to be small, the assumption which is required by perturbation method. Since in quark potential case it is not so the use of perturbation method looks very dubious.

Let us suggest that the static quark potential has vector and scalar property of Lorentz transform:

$$V_{NR}(r) = V_V(r) + V_S(r). \tag{1}$$

Index "v" means that potential is a 4-th component of operator $\widehat{p}_\mu$, index "s" means that the potential is scalar.

Following many authors we assume the admixture of vector-scalar screened potential

$$V_V(r) = V_{OGE} + \varepsilon \cdot V_{Conf}, \tag{2}$$

$$V_S(r) = (1-\varepsilon) \cdot V_{Conf}, \tag{3}$$

where

$V_{OGE}-$ is the one gluon exchange potential,

$V_{Conf}-$ is the confinement part of the potential.

where ε is mixing constant. Here the Lorentz nature of the one-gluon and the confining potential is different, the one-gluon potential being totally vector while the confining potential is a vector-scalar mixture. This choice seems to be reasonable since non-perturbative vertex corrections are important at small q^2 i.e. large distances and naturally are coupled only with the long-range term in potential. Very interesting rewiew was done in the work of Nora Brambilla conserning the choice of the interaction potential in [3].

In our case the choice of potential itself is dictated by the consideration of most accurate description of averaged mass spectrum, and here Chikovani-Jenkovszky-Paccanoni (CJP) type potential [4].

Our approach is based on the model of a non-perturbative gluon propagator, which was recently proposed by Chikovani, Jenkovszky and Paccanoni (CJP) [4]. In this model the propagator has the form

$$D\left(q^2\right) = \frac{c}{(q^2-\mu^2)} - \frac{1}{q^2-M^2} \tag{4}$$

and the corresponding potential assumes the form [4]

$$V(r) = \frac{g^2}{6\pi\mu}\left(1-\exp\left[-\mu r\right]\right) - \frac{16\pi}{25}\frac{\exp\left[-Mr\right]}{r\cdot\ln\left(b+1/\left(\Lambda r\right)^2\right)}. \tag{5}$$

In fact for accelerating numerical calculations we use simple form $\frac{\alpha_s}{r}$ for one-gluon-exchange type term with QCD-like of asymptotic freedom

$$\alpha_s(r) = \frac{12\pi}{33-2N}\cdot\frac{1}{\ln\left(1/\left(\widetilde{\Lambda}r\right)^2\right)}, \tag{6}$$

where $\widetilde{\Lambda}$ is taken to be equal to $\widetilde{\Lambda} = 0.14\ GeV$. The initial value of α_s was defined via calculating $q\overline{q}-$ masses.

At the finally, we present scalar and vector parts of the potential (2,3) in form

$$V_{OGE}(r) = -\frac{\alpha_S}{r}, \tag{7}$$

$$V_{Conf}(r) = \frac{g^2}{6\pi}\cdot\frac{(1-e^{-\mu r})}{\mu} \tag{8}$$

Recently screened potential of type (7,8) was successfully applied to a description of spin-averaged meson and baryon mass-spectra [5]. Recently, the study of the quark confining potential from QCD has been well developed. For example, the consideration of the gluon condense may give the confining potential a theoretical description. By considering the loop-diagrams of fermion in the lattice gauge calculation, E. Laermann et al [6] showed that the quark confinement potential goes lower than the linear confinement potential, when **r**, the distance between quarks, becomes larger. It reflects the screening between valence quarks and quark sea enhances, consequently, it suppresses the strength of color confinement between valance quarks. According to quantum mechanics, this feature may reduce the energies of highly excited baryons and mesons and help us to understand the experimental data.

Timo Thonhauser [7] approached the problem of the screened potential from opposite site, namely of Few-Body approach to baryon structure. Few-Body potential for baryons usually has confinement [8] part plus screened part. Very detailed description of nucleon-nucleon interaction is obtained within this approach. Thonhauser applied this potential to meson structure and obtained quite reasonable values of averaged meson masses. Unfortunately he did not report the results concerning fine splitting. The question of comparison of two approaches needs further study.

In addition, (see e. g. Gerasimov [9]) the spin-orbit term has to be of short range, as is indicated by quantum chromodinamics (QCD), the condition which is evidently satisfied by CJP potential.

In what follows we shall use the screened potential [4], which proved to be very good in describing the spin-averaged mass-spectrum of both bosons and baryons as quark systems [4],[10]-[13] and which secures the necessary fall-of the spin-dependent forces.

Let us start with two-body Fermi-Breit equation. We shall use nuclear system of units $\hbar = c = 1$,$1GeV = \frac{5.068}{1Fm}$. The Hamiltonian of the system has the form:

$$\widehat{H} = \widehat{H}_0 + \widehat{W}, \tag{9}$$

where

$$\widehat{H}_0 = -\frac{1}{2m}\triangle + \left(-\frac{\alpha_S}{r} + \frac{g^2}{6\pi}\frac{(1-e^{-\mu r})}{\mu}\right), \tag{10}$$

m is the reduced mass,

$$\widehat{W} = \widehat{H}_{LS} + \widehat{H}_{ST}, \tag{11}$$

where spin-dependent potentials are given by [1]:
• Spin-orbit interaction

$$\begin{array}{c} \widehat{H}_{LS} = \frac{1}{4m_1^2m_2^2}\frac{1}{r}\{\left[\left((m_1+m_2)^2+2m_1m_2\right)\overrightarrow{L}\cdot\overrightarrow{S_+} + \left(m_2^2-m_1^2\right)\overrightarrow{L}\cdot\overrightarrow{S_-}\right]\frac{dV_V}{dr} - \\ -\left[\left(m_1^2+m_2^2\right)\overrightarrow{L}\cdot\overrightarrow{S_+} + \left(m_2^2-m_1^2\right)\overrightarrow{L}\cdot\overrightarrow{S_-}\right]\frac{dV_S}{dr}\}, \\ \overrightarrow{L} = \overrightarrow{r}\times\overrightarrow{p}, \qquad \overrightarrow{S_+} \equiv \overrightarrow{S_1} + \overrightarrow{S_2}, \qquad \overrightarrow{S_-} \equiv \overrightarrow{S_1} - \overrightarrow{S_2}, \\ \overrightarrow{L}\cdot\overrightarrow{S} = \frac{1}{2}\left[j\left(j+1\right) - l\left(l+1\right) - S\left(S+1\right)\right]. \end{array} \tag{12}$$

Tensor terms

$$\begin{array}{c} \widehat{H}_T = \frac{1}{12m_1m_2}\left[\frac{1}{r}\frac{dV_V}{dr} - \frac{d^2V_V}{dr^2}\right]\cdot S_{12}, \\ S_{12} = \frac{4}{(2l+3)(2l-1)}\left[\overrightarrow{S}^2\cdot\overrightarrow{L}^2 - \frac{3}{2}\overrightarrow{L}\cdot\overrightarrow{S} - 3\left(\overrightarrow{L}\cdot\overrightarrow{S}\right)^2\right]. \end{array} \tag{13}$$

All notations are considered to be familiar and coincide with those which were used by Lucha and Schoberl in their well-known monograph [1].

Lucha and Shoberl [1] has indicated that the term $\frac{1}{r}\frac{dV_V}{dr}$ causes serious dimensional trouble because of particle falling on the center. But in a more consistent approach based on using Dirac equation we would have instead of previous term the following one $\frac{1}{E-V+mc^2}\frac{1}{r}\frac{dV_V}{dr}$, which for $r \to 0$ will behave like $\frac{1}{r^2}$ and the problem is removed. In matrix elements, we have calculated we obtained that, there are no essential differences between these two results. Previously we have studied these questions in [14].

The ether possibility to solve this problem is the regularisation of the potential [1]. The same idea is based on the using of $V_{reg} \propto \frac{1}{r+r_0}$ in the meaning of the regularized potential; where r_0- is free parameter, which has been obtain from the comparison with the experimental data. So in the case of $r = 0$ the problem of particle fooling is gone.

Based on this reason we believe that our numerical results are not up set by this problem.

However there are more "exact" QCD - motivated potentials, which are based on two loop back diagrams [15]. But by more detailed consideration it appears, that these accounts were carried out only for one-gluon exchange part. As races for systems considered by us, the very essential role is played by confinement. Besides use of this potential results in increase of number of parameters, with 1 in our case up to 3. Moreover analysis

the obtained results with two loopback potential show that of essential improvement of concurrence theoretical and experimental data is not observed.

In addition to above, indicated terms presented in Fermi-Breit Hamiltonian are the $\overrightarrow{S_1} \cdot \overrightarrow{S_2}$ (spin-spin) and relativistic correction terms (of p^4-order). Some authors (like [1]) indicated that these terms are important for calculating mass spectra, other authors (like [16]) argue that these results are purely constant. In our case we believe that for LS-mass differences they will not play an essential role.

Now, we consider Fermi-Breit equation

$$\left(-\frac{1}{2m}\triangle + \left(-\frac{\alpha_S}{r} + \frac{g^2}{6\pi}\frac{(1-e^{-\mu r})}{\mu}\right) + \widehat{W}\right)\Psi\left(\overrightarrow{r}\right) = \widehat{E}\Psi\left(\overrightarrow{r}\right). \tag{14}$$

There are certain difficulties in solving equation (14) in the case of spin-orbit-coupling. Using the potential (11) lead us to the appearance of term of order $1/r^3$. Usually authors use one of two possibilities:

a) use the perturbation method;

b) use the numerical computation.

Both of these possibilities are inferior: first one is unacceptable because in some cases the "fine" splitting turns out to be not so "fine" at all, being up to 50% contribution to the final mass, the second one needs introducing the cut-off parameter which is highly undesirable.

Here we suggest to use CI approach which was previously very successfully applied in atomic physics [2]. The essence of this approximation is that the total wave function $\Psi\left(\overrightarrow{r}\right)$ is expanded in set of eigen functions $\varphi_n$of the unperturbated Hamiltonian $\widehat{H_0}$, that is

$$\Psi\left(\overrightarrow{r}\right) = \sum a_n \varphi_n\left(\overrightarrow{r}\right). \tag{15}$$

After substituting (15) into (14) and using eigen value E_n^0 we obtain the system of linear equations for a_n which have to be truncated for reasonably large n.

$$\begin{array}{c} a_1\left(E - E_1^0 - W_{11}\right) - a_2 W_{12} - a_3 W_{13} - \ldots - a_n W_{1n} = 0 \\ -a_1 W_{21} + a_1\left(E - E_2^0 - W_{22}\right) - a_3 W_{23} - \ldots - a_n W_{2n} = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ -a_1 W_{n1} - a_2 W_{n2} - a_3 W_{n3} - \ldots + a_n\left(E - E_n^0 - W_{nn}\right) = 0 \end{array}, \tag{16}$$

where

$$W_{ij} = \langle\varphi_i|\,\widehat{W}\,|\varphi_j\rangle. \tag{17}$$

Both the basic functions φ_i and matrix elements W_{ij} are calculated numerically. Non-trivial solution there will be only if the determinant of this system vanishes can be solved by diagonalizing of the matrix for E. The system (16) is called CI. This procedure goes far outside of the perturbation method.

The CI approximation turned out to be extremely successful in atomic physics. In calculating atomic structure it allowed to increase the precision of calculating energy levels by one order. In the scattering processes it allowed to reveal fine resonance structure in scattering cross-sections due to formation of auto-ionizing states. So we expect that its applications will be even more important in strong interaction, where the perturbation

method is evidently not correct. The technique of application of CIA is quite complicated, since it needs to handle the matrices of large dimensions. In current work we used the code elaborated by O. Zatsarinny [17].

In this work we have applied the above described method for calculating $P-$ wave "fine"-splitting of $b\overline{b}-$, $c\overline{c}-$ and $u\overline{u}-$ systems to $^3P_0$, $^3P_1$ and $^3P_2$ levels. As well as for quarkonium constituted out of different masses like $s\overline{u}$, etc. In our case the corresponding operators $\widehat{H}_{LS}$, $\widehat{H}_T$ will be has the form

$$\widehat{W}_{LS} = \frac{1}{2m^2}\frac{1}{r}\left[3\frac{\alpha_S}{r^2} + (4\varepsilon - 1)\cdot\frac{g^2}{6\pi}e^{-\mu r}\right]\overrightarrow{L}\cdot\overrightarrow{S}, \tag{18}$$

$$\widehat{W}_T = \frac{1}{12m^2}\left[3\frac{\alpha_S}{r^3} + \left(\frac{1}{r} - \mu\right)\varepsilon\cdot\frac{g^2}{6\pi}e^{-\mu r}\right]S_{12}. \tag{19}$$

It is important that all parameters except ε are taken from [4],[13], where excellent description of bottomonium and charmonium spectra were obtained. Moreover as it was shown in [13] the same parameters gives good $\rho-$ meson trajectories. Actually the values $\frac{g^2}{6\pi} = 0.3\ GeV^2$, $\mu = 0.054\ GeV$ were taken and α_S was taken in accordance to QCD. The only adjustable parameter was ε. The experimental values are taken from [18]. As mentioned above all calculations were carried out numerically. Special code was constructed for this purpose. The calculations were extended to fifth order in (15) (see tables, rows 1-8), i.e. until the differences between the results did not go below several MeV level.

Table 1. $b\overline{b}$-system, $\alpha_s = 0.3$, $\varepsilon = 0.5$, $m_b = 5.05\ GeV$.

State	$\mathbf{\Delta M}_{TH}$	**order 1 MeV**	**order 2 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	21.32	21.23	21.3 ± 1.3
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	24.25	25.51	32.1 ± 2
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	45.57	46.74	53.4 ± 1.9
R		0.88	0.83	0.66
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$2^3P_2 - 2^3P_1$	–	17.72	13.3 ± 0.9
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_1 - 2^3P_0$	–	19.09	23.1 ± 1.1
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_2 - 2^3P_0$	–	36.81	36.4 ± 1.0
R		–	0.93	0.57
State	$\mathbf{\Delta M}_{TH}$	**order 3 MeV**	**order 4 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	21.20	21.19	21.3 ± 1.3
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	26.08	26.44	32.1 ± 2
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	47.29	47.63	53.4 ± 1.9
R		0.81	0.8	0.66
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$2^3P_2 - 2^3P_1$	17.61	17.58	13.3 ± 0.9
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_1 - 2^3P_0$	20.37	21	23.1 ± 1.1
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_2 - 2^3P_0$	37.98	38.58	36.4 ± 1.0
R		0.86	0.84	0.57

Table 2. $b\overline{b}$-system, $\alpha_s = 0.3$, $\varepsilon = 0.45$, $m_b = 5.05\ GeV$.

State	$\mathbf{\Delta M}_{TH}$	**order 1 MeV**	**order 2 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	20.00	19.92	21.3 ± 1.3
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	23.37	24.57	32.1 ± 2
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	43.37	44.49	53.4 ± 1.9
R		0.85	0.81	0.66
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$2^3P_2 - 2^3P_1$	–	16.68	13.3 ± 0.9
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_1 - 2^3P_0$	–	18.49	23.1 ± 1.1
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_2 - 2^3P_0$	–	35.25	36.4 ± 1.0
R		–	0.9	0.57

State	$\mathbf{\Delta M}_{TH}$	**order 3 MeV**	**order 4 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	19.89	19.87	21.3 ± 1.3
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	25.13	25.48	32.1 ± 2
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	45.02	45.36	53.4 ± 1.9
R		0.79	0.78	0.66
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$2^3P_2 - 2^3P_1$	16.76	16.65	13.3 ± 0.9
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_1 - 2^3P_0$	19.73	20.34	23.1 ± 1.1
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_2 - 2^3P_0$	36.38	36.95	36.4 ± 1.0
R		0.85	0.82	0.57

Table 3. $c\overline{c}$-system, $\alpha_s = 0.386$, $\varepsilon = 0.3$, $m_c = 1.675\ GeV$.

State	$\mathbf{\Delta M}_{TH}$	**order 1 MeV**	**order 2 MeV**	**order 3 MeV**	**order 4 MeV**
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	52.20	51.66	51.49	51.41
$\chi_{c1}\left(0^+\left(1^{++}\right)\right) - \chi_{c0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	69.31	77.27	81.55	84.51
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c2}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	121.51	128.93	133.04	135.93
R		0.75	0.66	0.63	0.61

State	$\mathbf{\Delta M}_{TH}$	**order 5 MeV**	**order 6 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	51.37	51.33	45.64 ± 0.25
$\chi_{c1}\left(0^+\left(1^{++}\right)\right) - \chi_{c0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	86.72	88.48	95.43 ± 1.12
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c2}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	138.09	139.82	141.07 ± 1.13
R		0.59	0.58	0.48

Table 4. $c\overline{c}$-system, $\alpha_s = 0.386$, $\varepsilon = 0.4$, $m_c = 1.675\ GeV$.

State	$\mathbf{\Delta M}_{TH}$	**order 1 MeV**	**order 2 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	66.81	66.17	45.64 ± 0.25
$\chi_{c1}\left(0^+\left(1^{++}\right)\right) - \chi_{c0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	79.36	88.57	95.43 ± 1.12
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c2}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	146.17	154.74	141.07 ± 1.13
R		0.84	0.75	0.48
State	$\mathbf{\Delta M}_{TH}$	**order 3 MeV**	**order 4 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	65.99	65.92	45.64 ± 0.25
$\chi_{c1}\left(0^+\left(1^{++}\right)\right) - \chi_{c0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	93.45	96.75	95.43 ± 1.12
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c2}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	159.45	162.67	141.07 ± 1.13
R		0.70	0.68	0.48

Table 5. $u\overline{u}$-system, $\alpha_s = 0.52$, $\varepsilon = 0.14$, $m_u = 0.33\ GeV$.

State	$\mathbf{\Delta M}_{TH}$	**order 1 MeV**	**order 2 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
$a_2\left(1^-\left(2^{++}\right)\right) - a_1\left(1^-\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	15.34	10.26	88.2 ± 40.7
$a_1\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	241.19	316.91	246.5 ± 40.9
$a_2\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	256.53	327.17	334.6 ± 1.6
R		0.063	0.032	0.357
State	$\mathbf{\Delta M}_{TH}$	**order 3 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**	
$a_2\left(1^-\left(2^{++}\right)\right) - a_1\left(1^-\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	8.57	88.2 ± 40.7	
$a_1\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	374.09	246.5 ± 40.9	
$a_2\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	382.66	334.6 ± 1.6	
R		0.023	0.357	

Table 6. $u\overline{u}$-system, $\alpha_s = 0.52$, $\varepsilon = 0.145$, $m_u = 0.33\ GeV$.

State	$\mathbf{\Delta M}_{TH}$	**order 1 MeV**	**order 2 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
$a_2\left(1^-\left(2^{++}\right)\right) - a_1\left(1^-\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	24.41	19.15	88.2 ± 40.7
$a_1\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	247.52	324.96	246.5 ± 40.9
$a_2\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	271.92	344.10	334.6 ± 1.6
R		0.098	0.059	0.357
State	$\mathbf{\Delta M}_{TH}$	**order 3 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**	
$a_2\left(1^-\left(2^{++}\right)\right) - a_1\left(1^-\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	17.45	88.2 ± 40.7	
$a_1\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	383.24	246.5 ± 40.9	
$a_2\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	400.69	334.6 ± 1.6	
R		0.046	0.357	

Table 7. $s\overline{u}$-system, $\alpha_s = 0.421$, $\varepsilon = 0.1875$, $m_s = 0.5\ GeV, m_u = 0.33\ GeV$.

State	$\mathbf{\Delta M}_{TH}$	**order 1 MeV**	**order 2 MeV**	**order 3 MeV**	**order 4 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
?	$1^3P_2 - 1^3P_1$	43.16	40.23	39.29	38.87	–
?	$1^3P_1 - 1^3P_0$	187.78	230.62	259.86	282.63	–
?	$1^3P_2 - 1^3P_0$	230.94	270.85	299.15	321.50	–
R		0.23	0.17	0.15	0.14	–

Table 8. $s\overline{u}$-system, $\alpha_s = 0.421$, $\varepsilon = 0.2$, $m_s = 0.5\ GeV, m_u = 0.33\ GeV$.

State	$\mathbf{\Delta M}_{TH}$	**order 1 MeV**	**order 2 MeV**	**order 3 MeV**	$\mathbf{\Delta M}_{EXP}$[18] **MeV**
?	$1^3P_2 - 1^3P_1$	60.13	56.95	56.00	–
?	$1^3P_1 - 1^3P_0$	199.43	244.92	331.74	–
?	$1^3P_2 - 1^3P_0$	259.56	301.84	387.75	–
R		0.30	0.23	0.17	–

Let us make the following conclusions:

1. The results for heavy quarkonium are quite good for values $\varepsilon = 0.3 - 0.45$. Which is coincide with results obtained by Lai-Him Chan [19] For light quarkonium the results are worse, which means that relativistic effects have to be taken into account more carefully. We believe, that the difference between ε in case of $b\overline{b}$ and $u\overline{u}$ system exactly reflects this fact.

2. The value of indicates that confinement has prevailingly scalar character. This conclusion does not contradict other authors [20].

3. As it follows from (18) at $\varepsilon = 0.25$ $(4\varepsilon - 1) = 0$ the contribution of confinement vanishes totally. May be exactly this circumstances was the reason that some authors stated the pure one-gluon character of LS-splitting.

4. The first column in tables corresponds to pure perturbation approach. It is clearly seen that this approach gives only rough qualitative estimate, but the results are drastically improving with switch on the CIA expansion. We believe that the using of CIA approach in quark physics will has bright future. Very convenient for fine-splitting characteristic is the coefficient

$$R = \frac{M\left({}^3P_2\right) - M\left({}^3P_1\right)}{M\left({}^3P_1\right) - M\left({}^3P_0\right)}. \tag{20}$$

As cited in [1] the experimental values of this parameter are $R = 0.66$ for $b\overline{b}$, $R = 0.48$ for $c\overline{c}$ and also $R = 0.21$ for $u\overline{u}$. It is interesting to note that firstly this parameter was introduced in atomic physics [21] where its value was established for atoms with two outer electrons (like Mg) as being $R = 2$. Since the fine mass splitting in atoms is totally due to one-photon exchange this value gives good indication as to the nature of the Lorentz character of $q\overline{q}-$ potential. As our calculations show the one-gluon term is totally vector while non-perturbative confinement term is of about 75-80% scalar.

Confinement gives 20-25% plus to the results, obtained with using only the one gluon exchange term, for the description of the $LS-$, $ST-$ interaction. Finally we obtain description of fine-effects at the quark systems with the precision 80-90%.

These conclusions confirms with the qualitative estimation of Lucha and Shöberl [1]. This result approximately confirms with the conclusion of Franzini [16] in the part, that the confinement is pure scalar. We show that the results are very sensitive to the exact value of ε and small change of ε can destroy the agreement with the experiment what actually occurred in the icase of Franzini.

We believe that exactly this feature will be successfully exploited in near future not only in hadronic structure, but also explaining resonances in $\pi\pi-$ or $\pi N-$ resonances.

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